



Designing Augmented Reality-Integrated Digital Teaching Materials Using TopWorksheets for Enhancing Mathematical Thinking Skills in Prism Learning

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Abstract

This study aims to develop Augmented Reality (AR)-based digital instructional materials using TopWorksheets that are valid, practical, and effective in improving students' mathematical thinking skills regarding prisms. The novelty of this research lies in the instructional design that explicitly integrates stages of mathematical thinking into the structure of interactive worksheets based on TopWorksheets combined with AR visualization a combination that has not been extensively studied in existing AR geometry applications or digital worksheets. This Research and Development (R&D) study followed the ADDIE model. The product was validated by three experts (content, construct, ICT, and language), its practicality was assessed through a student response questionnaire, and it was pilot-tested using a single-group pretest-posttest design over three sessions involving 19 ninth-grade students at SMP Negeri 6 Palembang. Data were collected through expert validation sheets, practicality questionnaires, pretest-posttest scores, observations, and interviews. The results showed a validity score of 84.31% (highly valid) and a practicality score of 86.72% (highly practical). The average N-Gain of 0.47 (moderate category) indicated a moderate pretest-posttest improvement in mathematical thinking skills. A qualitative analysis of selected cases representing variations in students' abilities showed a shift from misconceptions toward more accurate conceptual understanding, particularly in the "conjecturing" and "convincing" indicators, although difficulties in distinguishing between the height of a base and the height of a prism persisted in some groups. Non-cognitive factors, such as engagement and interest, also influenced individual learning outcomes. The instructional materials developed are suitable for use in mathematics education, with the main limitations being a single-group design and a small sample size meaning the results cannot yet be generalized and the need for refinement before wider implementation.

Keywords: Augmented reality; Digital teaching materials; TopWorksheets; Mathematical thinking skills; Prism

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INTRODUCTION

Mathematical thinking skills are one of the essential competencies that need to be developed in 21st-century mathematics education (Susanti et al., 2025). This ability is not only related to computational skills, but also encompasses higher-order cognitive processes such as analyzing, reasoning, problem-solving, and generalizing patterns (Khairunnisa et al., 2022). Indicators of mathematical thinking include specializing, generalizing, conjecturing, and convincing (Lestari et al., 2022). Mathematical thinking skills serve as a crucial foundation for students to understand concepts in depth, establish connections between concepts (Kolar & Hodnik, 2021), and apply mathematics in various real-life situations (Firmasari et al., 2022).

However, students' mathematical thinking skills remain relatively low (Singh et al., 2024), particularly in geometry (Tobing et al., 2022); this is the rationale behind this study.

Many students struggle to connect representations of three-dimensional shapes to mathematical forms (Yahya & Hershkowitz, 2024), identify the elements of a prism, visualize three-dimensional objects (Hwang et al., 2023), and apply the concepts of volume and surface area to problem-solving (Chiphambo & Mtsi, 2021). These difficulties arise because abstract mathematical concepts (Velez et al., 2023) require spatial visualization that has not yet developed optimally in most students (Mjenda et al., 2023), resulting in students' difficulty in constructing mental representations of three-dimensional shapes (Sablé-Meyer et al., 2022), understanding the relationships among geometric elements (Mudhefi et al., 2024), and drawing logical mathematical conclusions (Säfsström et al., 2024).

These limited abilities are also influenced by teaching methods that remain teacher-centered (Maskar & Herman, 2024), limited instructional materials and media (Kaplar et al., 2021), and the suboptimal use of technology (Laksmiwati et al., 2023). The media used are generally still static and two-dimensional, thus failing to support the dynamic visualization of three-dimensional shapes (Martins et al., 2023). As a result, geometry instruction tends to focus on memorizing formulas without conceptual understanding, causing students to struggle with contextual problems or different visual representations highlighting the urgent need for innovative digital instructional materials that make optimal use of technology (Tuwoso et al., 2021; Sari et al., 2025).

Advances in educational technology open up opportunities for more interactive learning experiences (Elvina et al., 2024). One such technology is Augmented Reality (AR), which displays three-dimensional objects realistically and allows them to be viewed from various angles (Aditya et al., 2022; Malta et al., 2023), thereby effectively facilitating the visualization of three-dimensional shapes (Koparan et al., 2023) and helping students understand the structure and properties of prisms (Rohendi et al., 2025). In addition to supporting specialization and generalization through direct visualization of objects, AR also has the potential to facilitate higher-order reasoning: its ability to quickly present and manipulate various prism shapes provides students with a foundation for comparing patterns across variations when making conjectures, while three-dimensional visualizations that can be rotated from various angles provide visual evidence that students can refer to when testing their mathematical arguments (convincing), thereby allowing AR to serve as more than just a spatial aid (Del Cerro Velázquez & Morales Méndez, 2021; Elsayed & Al-Najrani, 2021).

On the other hand, e-LKPDs provide teachers with the opportunity to offer flexible, structured learning activities that can assess students' thought processes (Huda et al., 2025). One such example is TopWorksheets, a web-based platform for creating interactive worksheets with automatic grading and performance tracking (Mahardika et al., 2023). However, most e-LKPDs are still passive and have not been integrated with visualization tools that support in-depth concept exploration. Yet, the development of mathematical thinking requires active learning experiences that encourage students to understand problems, create representations, devise strategies, and draw conclusions (Tanujaya et al., 2021).

Given these circumstances, the integration of TopWorksheets-based digital instructional materials with AR presents a potential solution. TopWorksheets guides learning activities and tracks student progress (Sodikin et al., 2025), while AR provides realistic visualizations of mathematical objects (Schutera et al., 2021), facilitating the understanding of concepts that are difficult to visualize conventionally (Radu & Schneider, 2023).

Several studies have shown that AR in mathematics education enhances students' conceptual understanding (Pahmi et al., 2023), motivation, and engagement (Çetintav & Yılmaz, 2023). However, most of these studies utilize AR as a standalone visualization tool, independent of the worksheet structure that guides students step-by-step through the four mathematical thinking processes, and have not been specifically designed to address certain geometric misconceptions such as errors regarding the base and height of prisms identified in the background of this study. Thus, the gap is not merely the limited number of studies

integrating AR with TopWorksheets, but rather the lack of designs that intentionally combine AR visualization with a structured worksheet sequence to guide students from “specializing” to “convincing” while simultaneously remediating these misconceptions.

TopWorksheets was chosen as the e-LKPD platform because it offers a combination of features not fully available on similar platforms. Compared to LiveWorksheets, Google Forms, and Quizizz, TopWorksheets allows students to access worksheets without needing an account a feature also offered by LiveWorksheets but not by Google Forms or Quizizz. In terms of grading, both TopWorksheets and Quizizz provide fully automated grading, while LiveWorksheets only supports it to a limited extent and Google Forms requires additional setup for automated grading to function. The most distinctive advantage of TopWorksheets is its ability to embed links or external AR objects directly into worksheets a feature not available in LiveWorksheets, Google Forms, or Quizizz. In terms of ease of access, all four platforms can be integrated via the web without requiring any additional installation. It is this combination of features that makes TopWorksheets the most suitable platform for this study.

Based on the above description, this study aims to develop TopWorksheets-based digital instructional materials integrated with AR that are valid, practical, and effective in enhancing students’ mathematical thinking skills regarding prisms, with the research question: How can these instructional materials be developed to be valid, practical, and effective? More specifically, the contribution of this study lies not only in a valid and practical product but also in an instructional sequence that combines AR visualization with structured worksheets to guide students step-by-step through the four indicators of mathematical thinking, while simultaneously addressing misconceptions regarding the base and height of prisms.

METHOD

Research Design

This study is a research and development (R&D) project that employs the ADDIE model, which consists of five stages: Analysis, Design, Development, Implementation, and Evaluation (Ding & Toran, 2025). This model was chosen because it provides a systematic development process for producing TopWorksheets-based digital instructional materials integrated with Augmented Reality (AR) that are valid, practical, and effective in improving students’ mathematical thinking skills regarding prisms.

Research Participants

The participants in this study were 19 ninth-grade students at SMP Negeri 6 Palembang, Indonesia, aged 14–15 years. The school and class were selected using purposive sampling based on the school’s willingness to participate, the alignment of the prism material with the curriculum and class schedule, and the researcher’s access to the school. Inclusion criteria included: (a) being enrolled as a 9th-grade student scheduled to study the prism material; (b) having never used TopWorksheets-based digital instructional materials integrated with augmented reality (AR); and (c) being willing to participate with parental consent and the student’s own agreement. The pilot test was conducted in a single, complete class designated by the school to allow the product to be implemented in real-world learning conditions and to enable to gather information regarding implementation, student responses, practicality, and changes in learning outcomes. This sample size was deemed appropriate for a limited pilot test during the Implementation phase of product development.

ADDIE Development Process

The Analysis phase was conducted to identify learning needs, student characteristics, curriculum requirements, learning objectives, and difficulties with the prism material through a curriculum review and interviews with teachers. The Design phase resulted in a teaching material structure consisting of three topics the elements of a prism, surface area, and volume with an activity sequence of observing, exploring, discovering, and applying. During the Development phase, the initial prototype was validated by three expert validators: Dr. Budi

Mulyono, S.Pd., M.Sc., as an ICT expert; Dr. Sujinal Arifin, M.Pd., as a subject matter expert; and Bastomi, S.Pd., as a language expert. Validators were selected based on qualifications and experience relevant to the field being evaluated, as well as their involvement in teaching or instructional material development. Each validator assessed the content, construct, ICT, and language aspects using an expert validation questionnaire. The results of the assessment and the validators' recommendations were used to revise the product, followed by an initial practicality test via a student response questionnaire. The Implementation phase was conducted over three sessions covering all three topics, beginning with a pretest and ending with a posttest using the same instruments, and supported by classroom observations and brief interviews. The Evaluation phase was conducted formatively throughout the development process to guide product improvements and summatively after implementation to assess the product's validity, practicality, and outcomes of its use. The development procedures carried out including the stages, outputs, validation, revision, testing, and evaluation are presented in Figure 1.

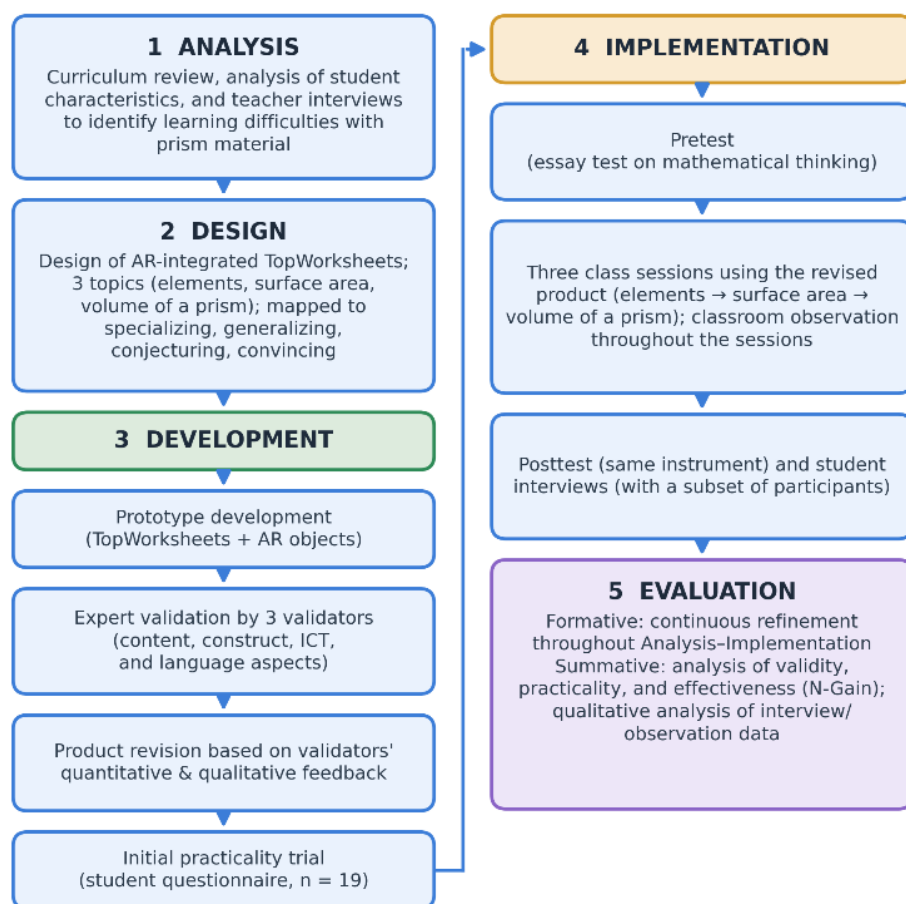


Figure 1. ADDIE research flow of the development and implementation of AR-integrated TopWorksheets

Instruments

Four instruments were used to collect quantitative and qualitative data, as summarized in Table 1.

Table 1. Summary of Research Instruments

Attribute	Expert Validation Questionnaire	Student Response Questionnaire	Pretest-Posttest	Combination of Interviews & Observations
– Objectives	To assess the validity of the product	To assess the practicality of the product	To measure the effectiveness/achievement of	To support /triangulate quantitative findings

Attribute	Expert Validation Questionnaire	Student Response Questionnaire	Pretest-Posttest	Combination of Interviews & Observations
– Dimensions/ Indicators	Content, construct, ICT, language	Ease of use, appeal, clarity of material, time efficiency, benefits/ feasibility	mathematical thinking Specializing, generalizing, conjecturing, convincing	Student engagement, learning experience, feasibility of instruction
– Number of Items	21 statements	16 statements	3 essay questions x 4 sub-questions	Semi-structured
– Respondents	3 expert validators	19 students	19 students	Selected students and teachers

The mathematical thinking skills test consists of three questions, with each subquestion scored on a scale of 0–3 using an analytical rubric. For the “specializing” indicator, the assessment ranges from irrelevant answers, partial information, one piece of complete information, to complete and accurate information that is both known and asked for. For “generalizing,” the criteria range from an incorrect process, an incorrect process and result, a correct process with an incorrect result, to a correct process and result. For “conjecturing,” the assessment ranges from guesses without reasoning, to guesses with insufficient reasoning, to accurate guesses supported by relevant reasoning. For “convincing,” the criteria range from incomplete or illogical proofs, to correct but incomplete proofs, to complete, logical, and correct proofs. The mathematical thinking skills test instrument is presented below.

Table 2. Mathematical Thinking Questions

Question	Question Context	Requested Task	Indicator
1	Surface area of a binder shaped like a right-angled triangular prism	– Write down the given and asked information – Calculate the surface area of the binder based on the given measurements – Conjecturing whether the amount of plastic needed would be exactly half if the binder’s height were cut in half – Proving the correctness of the conjecture in 1c and providing a rationale	Specializing Generalizing Conjecturing Convincing
2	The volume of water in a rectangular prism-shaped can	– Write down the given and asked information – Determining the initial volume of water and the volume after half has been removed – Estimating the height of the water after adding 40 cm³ and explaining the reasoning – Calculate the water level after the addition and verify the accuracy of the estimate in part 2c	Specializing Generalizing Conjecturing Convincing
3	Changes in the dimensions of a triangular prism-shaped chocolate package	– Write down the given and asked information – Calculating the volume of the old package and the base side length of the new package (reduced by 20%) – Determining what must happen to the height of the prism so that the volume remains the same – Determining and proving whether the new box is longer or shorter, and calculating the percentage change	Specializing Generalizing Conjecturing Convincing

Data Analysis

All data were analyzed using descriptive quantitative and qualitative techniques to determine the validity, practicality, and effectiveness of the developed product. Validity data from the expert validation sheets were first analyzed by converting each validator's score into a percentage using the following formula.

$$V = \frac{f}{N} \times 100\%$$

where (V) is the final score as a percentage, (f) is the total score obtained, and (N) is the maximum score. The results were interpreted based on the validity categories in the table below.

Table 3. Validity Criteria

Validity Criteria	Validity Level Qualification
81% – 100%	Highly valid and usable without revision
61% – 80%	Valid and usable but requires minor revisions
41% – 60%	Fairly valid and use is not recommended
21% – 40%	Invalid and cannot be used
0% – 20%	Highly invalid and cannot be used

Next, practicality is calculated using the following formula

$$Vp = \frac{TSEp}{S - max} \times 100\%$$

where (Vp) is the practicality percentage, (TSEp) is the total score obtained, and (S-max) is the maximum score. The percentage obtained is then interpreted to determine the level of practicality of the developed instructional material. The criteria for interpreting the percentage are presented in the following table.

Table 4. Practicality Criteria

Practicality Criteria	Practicality Level Qualification
81% – 100%	Very practical and usable without revision
61% – 80%	Practical and usable but requires minor revisions
41% – 60%	Fairly practical and use is not recommended
21% – 40%	Not practical and cannot be used
0% – 20%	Very impractical and cannot be used

Effectiveness is analyzed based on the difference between pretest and posttest scores using the N-Gain formula.

$$N - Gain = \frac{\text{posttest score} - \text{pretest score}}{\text{maximum score} - \text{pretest score}}$$

The N-Gain calculation results are then categorized based on the following criteria:

Table 5. Interpretation N-Gain

N-Gain	Category
$g > 0,7$	High
$0,3 < g < 0,7$	Moderate
$g < 0,3$	Low

Qualitative data obtained from validator comments, interview results, and observation findings were analyzed descriptively through a process of data reduction, data presentation, interpretation, and drawing conclusions. Qualitative data was used to identify shortcomings in the instructional materials, gather information about user experiences, and determine which parts of the product needed improvement. Thus, quantitative and qualitative data were analyzed in an integrated manner to provide a comprehensive picture of the quality of the developed product.

RESULTS AND DISCUSSION

Analysis Phase

The analysis phase was conducted through discussions with mathematics teachers to understand the learning conditions, the materials used, learning resources, and students' difficulties with the prism unit. Based on the discussion results, the prism unit is included in the Geometry and Measurement unit of the Merdeka Curriculum Phase D. The following is a record of the discussion with the teachers.



Figure 2. Interview with Teachers

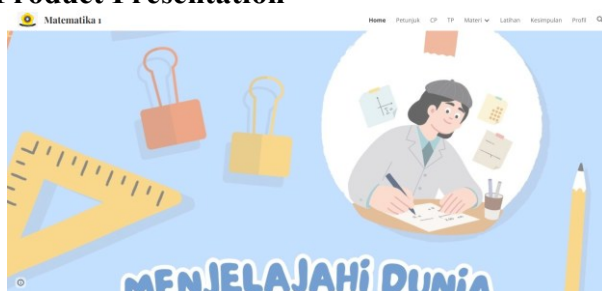
Teachers use textbooks as the primary learning resource; however, the content is still dominated by two-dimensional illustrations, explanations, and practice problems, limiting students' ability to explore prisms directly and leading them to memorize formulas without understanding the relationships between the prism's elements particularly distinguishing between the base height and the prism height when solving problems involving surface area and volume.

The discussion also highlighted the need for more engaging and interactive teaching materials so that students can observe, explore, discover relationships, and justify their answers. Based on these needs, digital instructional materials based on TopWorksheets and integrated with Augmented Reality (AR) were developed. TopWorksheets are used to provide interactive activities, while AR helps students explore prism objects in three dimensions so that the positions and sizes of the prism's elements particularly the base height and the prism height are easier to understand. The results of this analysis served as the basis for designing the instructional materials during the Design phase.

Design Phase

The Design Phase was carried out based on the results of the needs analysis by designing digital instructional materials that integrate TopWorksheets and Augmented Reality (AR) for prism-related content. The instructional materials consist of three parts: the elements of a prism, the surface area of a prism, and the volume of a prism, which are organized progressively through activities involving observation, exploration, identifying patterns and concepts, and problem-solving.

Product Presentation



Home Screen



TopWorksheets Activities

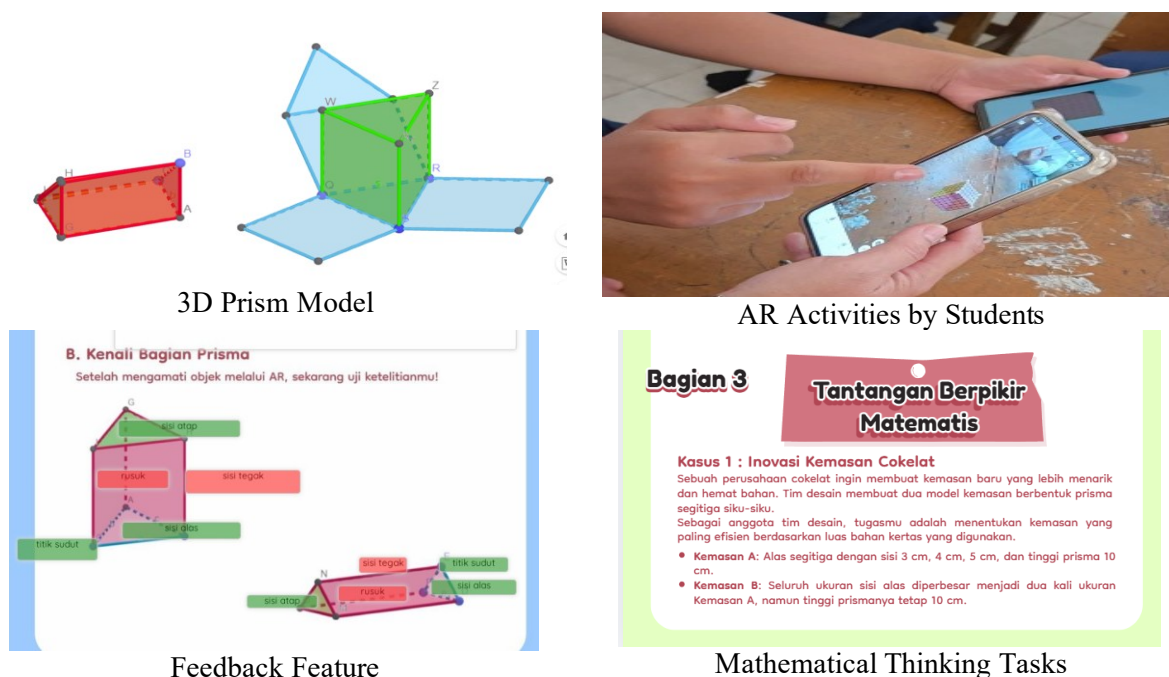


Figure 3. Presentation of the Product

The home screen serves as the main page that introduces the instructional materials, while the navigation page provides instructions for using the menu and access to the learning sections. Activities on TopWorksheets are used to present exercises and learning activities digitally. AR integration is demonstrated through a scanning process using students' devices, which generates a three-dimensional prism model via GeoGebra. The model includes visualizations of the net and the three-dimensional shape of the prism to support observation of the prism's characteristics and elements. The product also features automatic feedback that indicates whether a student's answer is correct or incorrect. In the Mathematical Thinking Challenges section, students are presented with contextual problems designed to develop the indicators of specializing, generalizing, conjecturing, and convincing. With these components, the product combines content presentation, digital activities, 3D visualizations, feedback, and mathematical thinking tasks into a single instructional resource.

To clarify the relationship between each product component and the prism concept and the developed indicators of mathematical thinking, Table 6 maps the product activities, AR features, prism concepts, indicators of mathematical thinking, and expected learner actions.

Table 6. Product Mapping

Product Activities	Digital and AR Features	Prism Concepts	Mathematical Thinking Indicators	Expected Student Actions
– Observing everyday prism-shaped objects (Elements Section)	Visualizing 3D prism models using AR	Faces, edges, vertices, bases, and lateral faces	Specializing	Identifying and naming prism elements using the AR model
– Exploring n-sided prism models (triangular to hexagonal prisms)	Interactive GeoGebra-based AR models that can be rotated and zoomed	Relationships among the numbers of faces ($n + 2$), edges ($3n$), and vertices ($2n$)	Generalizing	Discovering and formulating general relationships among the

Product Activities	Digital and AR Features	Prism Concepts	Mathematical Thinking Indicators	Expected Student Actions
– Exploring prism nets and lateral faces (Surface Area Section)	Interactive AR model showing the folding and unfolding of a prism net	Relationships among base area, base perimeter, prism height, and surface area	Specializing, Generalizing	elements of n-sided prisms Identifying the components of prism nets and deriving the formula for prism surface area
– Comparing alternative packaging designs (Packaging A and B)	AR models of two prism-shaped packaging designs for comparison	Surface-area-to-volume ratio in packaging design	Conjecturing	Formulating conjectures about packaging efficiency based on observed geometric relationships
– Exploring prism volume using unit cubes (Volume Section)	AR model illustrating arrangements of unit cubes inside a prism	Volume as the number of unit cubes and its relationship to base area and prism height	Specializing, Generalizing	Calculating the number of unit cubes and deriving the formula for prism volume
– Investigating the invariance of prism volume under rotation and repositioning	Interactive AR model of a prism that can be rotated and repositioned	Invariance of prism volume under rotation and translation	Convincing	Constructing mathematical arguments to explain why prism volume remains unchanged under rotation and translation
– Completing and submitting assignments through TopWorksheets	Automated grading with correct/incorrect feedback	Prism concepts addressed in the corresponding learning activities	Specializing, Generalizing, Conjecturing, Convincing	Reviewing and correcting answers based on feedback before proceeding to the next activity

In addition to conceptual understanding, the three sections are designed to develop mathematical thinking skills through the indicators of specializing, generalizing, conjecturing, and convincing. In the section on the elements of a prism, students identify and generalize patterns in the number of elements. In the section on surface area, students analyze changes in the size of chocolate packaging, identify patterns, make conjectures, and provide reasons for more efficient designs. In the volume unit, students compare the volumes of several prisms with different sizes and orientations and explain that changing the position does not alter the volume. Overall, TopWorksheets serves as an interactive activity space, while AR allows students to explore prisms in three dimensions.

Development Phase

The initial version of the TopWorksheets-based digital teaching materials integrated with Augmented Reality, which was designed during the Design phase, was subsequently validated by three expert validators Dr. Bodi Mulyono, S.Pd., M.Sc., Dr. Sujinal Arifin, M.Pd., and Bastomi, S.Pd. who assessed four aspects: content, construct, ICT, and language.

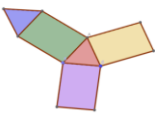
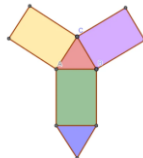
Table 7. Validation Results by Aspect

Aspect	Percentage	Category
Content	86,67%	Highly Valid
Construct	86,67%	Highly Valid
ICT	85,56%	Highly Valid
Language	78,33%	Valid
Overall Average	84,31%	Highly Valid

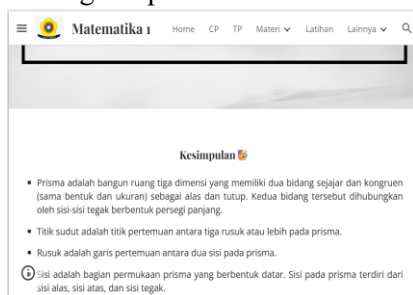
Based on the table above, overall the product achieved a validity percentage of 84.31%, which falls into the “highly valid and usable without revision” category. The content and construct aspects received the highest percentage (86.67%), indicating that the prism material presented aligns with the learning objectives, is systematically organized, and effectively facilitates students in discovering concepts and practicing mathematical thinking indicators. The ICT aspect received 85.56%, indicating that the use of the TopWorksheets platform and AR visualization was deemed sufficiently supportive of understanding prism concepts. Meanwhile, the language aspect received the lowest percentage (78.33%) and falls into the “valid but requires minor revisions” category, particularly regarding the clarity of terminology and the potential for ambiguity in question instructions.

Although the quantitative validation results indicated the “highly valid” category, the researchers still made revisions based on qualitative suggestions from the three validators to improve the product’s quality before implementation. The three validators provided a number of suggestions for improvement, which were addressed through product revisions, as presented in the following table.

Table 8. Validators' Comments/Suggestions and Revision Decisions

No	Comments/Suggestions	Revision Decision
1.	The validator provided suggestions for improving the net diagram of the prism B. Logika Penjumlahan 	The shape and position of the vertical sides were revised to appear proportional and clearly rectangular, making the representation of the triangular prism’s net more accurate and easier to understand. B. Logika Penjumlahan 
2.	The validator provided suggestions to add an explanation of the shape of a Scout tent Mari kita uji pemahamanmu! Jangan khawatir, kita akan mengerjakannya secara bertahap. Ikuti langkah-langkah di bawah ini untuk menyelesaikan tantangan pertama. Tantangan: Menghitung Luas Tenda Pramuka Sebuah tenda pramuka berbentuk Prisma Segitiga Siku-siku memiliki ukuran sebagai berikut: • Alas segitiga (a): 3 m • Tinggi segitiga (b): 4 m • Sisi miring segitiga (c): 5 m • Tinggi tenda/prisma (t): 6 m Langkah 1: Hitung Luas Alas (La)	An explanation was added to clarify that a scout tent has two triangular sides connected by three rectangular sides, so that students can relate the shape of the tent to the structure of a right-angled triangular prism before determining its surface area.

3. The validator suggests adding descriptions and diagrams of quadrangular prisms in addition to rectangular prisms



Mari kita uji pemahamanmu! Jangan khawatir, kita akan mengerjakannya secara bertahap. Ikuti langkah-langkah di bawah ini untuk menyelesaikan tantangan pertama.

Tantangan: Menghitung Luas Tenda Pramuka

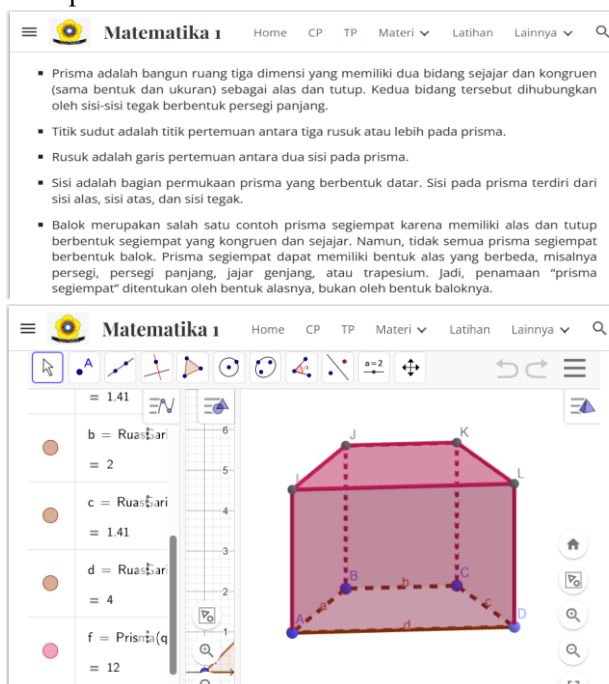
Sebuah tenda pramuka berbentuk prisma segitiga. Tenda ini memiliki dua sisi berbentuk segitiga, yaitu di bagian depan dan belakang. Kedua segitiga tersebut dihubungkan oleh tiga sisi berbentuk persegi panjang. Salah satu sisi persegi panjang berada di bagian bawah sebagai alas tenda, sedangkan dua sisi persegi panjang lainnya membentuk dinding tenda yang menjadi ruang untuk duduk dan beraktivitas di dalamnya.

Perhatikan ukuran tenda berikut:

- Alas segitiga (a): 3 m
- Tinggi segitiga (b): 4 m
- Sisi miring segitiga (c): 5 m
- Tinggi tenda/prisma (t): 6 m

1 Langkah 1: Hitung Luas Alas (L_a)

An explanation has been added stating that a rectangular prism is one example of a quadrangular prism with congruent and parallel quadrilateral bases. Quadrangular prisms can also have bases in the shape of squares, parallelograms, or trapezoids.



Next, preliminary data on usability were obtained through a questionnaire completed by students after using the revised digital instructional materials. The questionnaire used a four-point scale: Strongly Agree, Agree, Disagree, and Strongly Disagree. The questionnaire results were interpreted based on the criteria in the following table.

Table 9. Results of the Student Usability Questionnaire by Aspect

Aspect	Percentage	Category
Ease of Use	82,92%	Very Practical
Appeal	86,67%	Very Practical
Clarity of Content	87,19%	Very Practical
Time Efficiency	88,33%	Very Practical
Benefits and feasibility	88,33%	Very Practical
Overall Average	86,72%	Very Practical

Based on the table above, all aspects received the “very practical” category, with percentages ranging from 82.92% (ease of use) to 88.33% (time efficiency, benefits, and feasibility). Overall, the TopWorksheets-based digital instructional materials integrated with Augmented Reality achieved an average practicality percentage of 86.72%, which falls into the

“very practical and usable without revision” category. These results indicate that students found the instructional materials easy to access and use, engaging, helpful in understanding prism concepts, and efficient in terms of learning time, thereby supporting the product’s suitability for use in the Implementation phase.

Based on the validation and revision results, as well as the initial practicality survey results from the students, the final product digital instructional materials based on TopWorksheets integrated with Augmented Reality achieved an average validity percentage of 84.31% (highly valid category) and an average practicality percentage of 86.72% (highly practical category). The revised product was subsequently deemed suitable for use in the Implementation phase.

Implementation Phase

The final product, which has been validated and deemed practical, was subsequently implemented in a Grade 9 mathematics lesson on prisms, covering three topics: the elements, surface area, and volume of prisms. Students took a pretest before and a posttest after the entire learning process, using the same instrument three essay questions, each with four sub-questions (a–d) that measure specializing, generalizing, conjecturing, and convincing.

In summary, during the first session, all groups successfully identified the patterns for the number of faces, edges, and vertices of an n -sided prism, although some groups were still somewhat inaccurate in their descriptive exploration of the prism’s elements and had difficulty distinguishing the base from the lateral faces. In the second session, all groups successfully derived the formula for the surface area of a prism, although errors in the order of operations were found during the guided practice, and there were variations in ability during the conjecturing and convincing stages of the packaging design challenge. In the third session, most groups understood the relationship between base area $\times$ height = volume, but misconceptions regarding the base height and the prism height persisted through two distinct error pathways, and errors were also found in selecting the correct multiplier for volume calculations.

An error occurred in the section exploring base height versus prism height, specifically regarding quadrangular and triangular prisms. Group 6 incorrectly identified the shape of the second prism’s base as a rectangle, when it should have been a triangle; consequently, their explanation regarding the base height and prism height was also incorrect. Group 3, on the other hand, correctly identified both base shapes but still reached an incorrect conclusion because they assumed the base height and the prism height were the same, reasoning that “the sums are the same.” These two groups took two different wrong paths, yet both led to the same problem: a misconception regarding the base height and the prism height, which remains a challenge and requires further attention, regardless of the groups’ ability to identify the prism’s base shapes. This can be seen in the following figure.

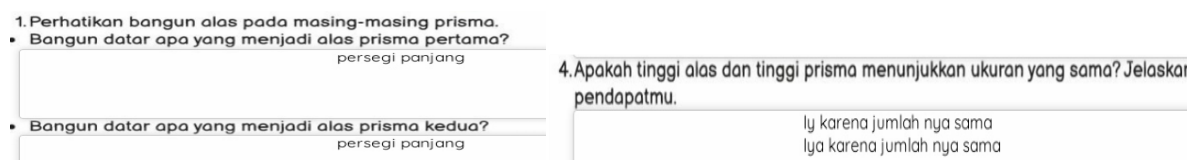


Figure 4. Misconceptions Regarding Base Height and Prism Height

The key findings from the three implementation sessions are summarized in the following table, covering the targeted concepts, observed achievements, and dominant misconceptions for each session.

Table 10. Synthesis of Findings from Meetings 1 through 3

Session	Targeted Concepts	Observed Outcomes	Dominant Misconceptions
1	Identification of the elements of a prism	All groups successfully determined the patterns	Confusion in distinguishing between the shape of the base and

2	(faces, edges, vertices, base, top, lateral faces) and the general pattern of an n-sided prism	for the number of faces, edges, and vertices of n-sided prisms (including decagons)	the shape of the vertical sides; the description of the number of bases/lids was inaccurate in some groups
	Formula for the surface area of a prism using a net; packaging design efficiency	All groups correctly derived the surface area formula and succeeded in the “specializing” stage (packages A & B)	Errors in the order of mathematical operations (multiplication before addition); some non-mathematical reasoning in conjectures; convincing arguments were not consistently developed
3	Volume of a prism using unit cubes; distinguishing between base height and prism height	Most groups understood the relationship: base area $\times$ height = volume	Misconceptions regarding base height vs. prism height (through two different error pathways); errors in selecting the correct multiplier (side length vs. prism height) in calculations

In general, the exploration process using AR in all three sessions went well, although variations in achievement and misconceptions were still found in some groups. To examine the impact on students’ mathematical thinking skills, the following presents the quantitative pretest-posttest results.

Table 11. Descriptive Statistics of Pretest and Posttest Results

Statistics	Pretest	Posttest
Average Score	24,42	56,29
Average Category	Very Low	Moderate
Highest Score	63,89	100
Lowest Value	5,56	5,56
Standard Deviation	17,14	28,55
Median	19,44	55,56
1st quartile	13,89	44,44
3rd quartile	30,56	76,39

Based on the pretest and posttest results, students’ average scores increased from 24.42 (very low) to 56.29 (moderate). This increase indicates a change in mathematical thinking skills following the use of the instructional materials. However, the improvement was not uniform, as evidenced by the increased variability in scores from the pretest to the posttest. The posttest average, which remains in the moderate category, also indicates that students’ achievement has not yet reached a high level overall. To assess the magnitude of the improvement more precisely, an N-Gain analysis was conducted, as presented in the following section.

Table 12. N-Gain Values

Statistics	Number of Students	Percentage
High ($g \geq 0,7$)	5	26,3%
Moderate ($0,3 \leq g < 0,7$)	9	47,4%
Low ($g < 0,3$)	5	26,3%
Overall Average N-Gain	0,47 (Moderate)	

Based on the table above, the majority of students (47.4%) showed a moderate improvement in mathematical thinking skills, while those in the high and low categories accounted for 26.3% each. The average N-Gain of 0.47 falls into the moderate category, indicating an improvement in mathematical thinking skills following the use of the instructional materials, although student achievement varied. To further explore these findings, an analysis was conducted on the written responses from the pretest and posttest based on three subjects purposively selected from the high, moderate, and low N-Gain categories. This

selection was intended to illustrate the variation in student achievement, not to statistically represent the class. Subject AA was selected from the high category, DZ from the moderate category, and RB from the low category because they did not show any improvement. The analysis was conducted for each mathematical thinking indicator in accordance with the structure of the test instrument.

To test the significance of the change in scores, a Shapiro-Wilk normality test was conducted on the difference between pretest and posttest scores. The results showed that the data were normally distributed (statistic = 0.911; $p = 0.077 > 0.05$), so a paired t-test could be used. The test results showed a significant difference in scores between the pretest and posttest, $t(18) = 7.82$, $p < 0.001$, with a mean difference of 31.87 (95% CI: 23.31–40.44) and Cohen's $d = 1.79$. These values indicate a large difference in scores following the use of the instructional materials. However, because the study used a one-group pretest-posttest design without a comparison group, these results indicate a significant change in scores after implementation but cannot yet confirm that this change was entirely due to the instructional materials.

Because mathematical thinking skills are multidimensional, the analysis of total scores is supplemented by an analysis based on each indicator. The table below presents the average pretest and posttest scores as well as the N-Gain for each indicator of mathematical thinking skills.

Table 13. Pretest-Posttest Results by Mathematical Thinking Indicator

Indicator	Pretest		Posttest		N-Gain	N-Gain Categories
	Mean	SD	Mean	SD		
Specializing	30,99	12,05	58,48	23,67	0,43	Moderate
Generalizing	35,09	23,50	65,50	30,52	0,55	Moderate
Conjecturing	17,54	20,05	53,80	32,77	0,50	Moderate
Convincing	14,04	20,57	47,37	31,88	0,44	Moderate

Based on Table 13, all indicators showed improvement with an N-Gain in the moderate range, specifically 0.43–0.55. The highest N-Gain was observed in “generalizing” (0.55), while the lowest was in “specializing” (0.43). “Conjecturing” and “convincing” had lower pretest averages than “specializing” and “generalizing,” but both showed improvement on the posttest. These findings indicate that improvements occurred across all indicators, although the extent of improvement varied among them.

Specializing and Generalizing

On the “specializing” indicator, Subject AA showed improvement, moving from writing known and asked information in a fairly adequate manner on the pretest to doing so more explicitly and systematically on the posttest. Subject DZ exhibited the same pattern on both tests, namely writing only some of the given information without including important details regarding the change in water volume, while Subject RB, on both tests, merely copied the wording of the question without separating the given and asked parts and did not proceed to the next sub-question. Regarding the generalizing indicator, the ability of Subjects AA and DZ to apply general formulas for the surface area and volume of prisms was relatively good from the pretest onward; the most noticeable improvement was observed in the relevant generalizing sub-question, which neither subject attempted successfully on the pretest but both were able to solve correctly on the posttest, indicating an improved ability to apply prism formulas to more complex problem contexts.

Conjecturing

For this indicator, subject AA demonstrated a change in understanding between the pretest and posttest. On the pretest, AA agreed with the conjecture that reducing the binder's height by half would make the required plastic area exactly half of the original. This response indicated a misconception because the surface area of a prism does not change linearly based solely on a change in height. On the posttest, AA answered that a change in height does not

directly determine the proportion of the surface area because the surface area is also influenced by the size of the base. This indicates a more accurate understanding of the concept.

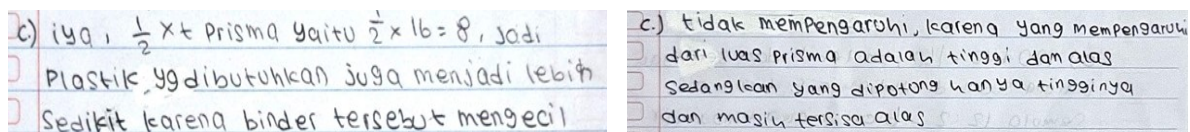


Figure 5. Subject AA's Answers on the Conjecturing Indicator

Subject DZ was already able to make accurate predictions as early as the pretest. On the pretest, DZ stated that Nisa's prediction was incorrect because the surface area did not become half that is, 300 is not half of 540. This answer indicates that DZ was able to use calculation results to support his prediction. In the posttest, this ability appeared to have developed further. DZ explained that the surface area is influenced by the base area, perimeter, and height of the prism, whereas the part that was cut off was only the height. This answer shows that DZ not only made a conjecture but also provided a rationale based on the components that influence the surface area of a prism. Thus, DZ's conjecturing ability was evident in both tests, with a more conceptual and comprehensive rationale in the posttest.

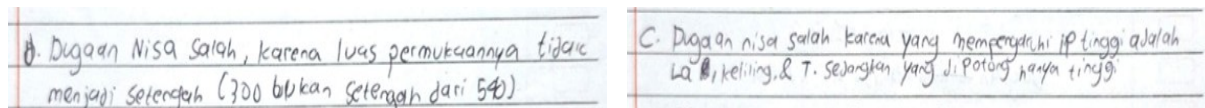


Figure 6. Subject DZ's Answer on the Conjecturing Indicator

Convincing

In the pretest, Subject AA provided reasoning based on his understanding without supporting calculations, and even reinforced the misconception from the previous section. In the posttest, he was able to provide mathematical proof by recalculating the surface area under the new conditions and comparing it to his initial conjecture, demonstrating improved convincing ability.

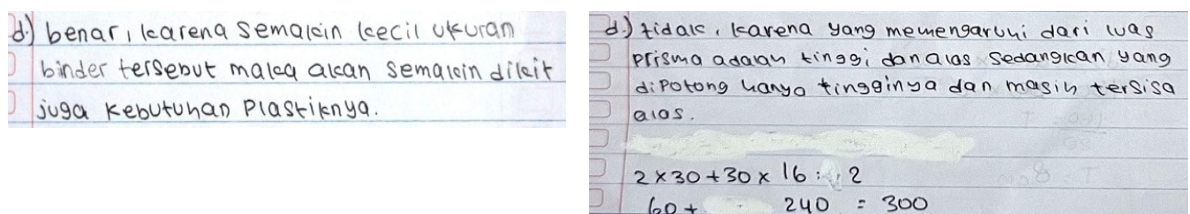


Figure 7. Subject AA's Responses to the "Convincing" Indicator

Subject DZ demonstrated progress in the sequence of his reasoning between the pretest and posttest. In the pretest, DZ tended to prove a statement through calculation first, then draw a conjecture based on the results obtained. Meanwhile, in the posttest, DZ was already able to make a conjecture first before performing mathematical proof. This change indicates that DZ's convincing ability has developed, as he no longer relies solely on calculations to reach a conclusion but is able to propose a conjecture and subsequently validate it through mathematical proof.

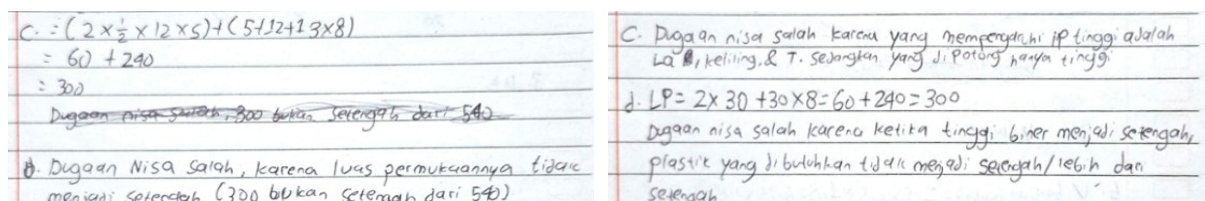


Figure 8. Subject DZ's Answers on the "Convincing" Indicator

In addition to these patterns in written responses, the achievements of these three students can also be understood through observations made during the learning process and brief

interviews conducted to explore the non-cognitive factors that also influenced their learning outcomes. The observation results showed that subject AA appeared very enthusiastic throughout the learning process. When interviewed about any shortcomings in the instructional materials used, AA stated that he did not perceive any shortcomings, as he found learning with those materials to be exciting. This enthusiasm persisted even though he had previously experienced difficulty distinguishing between the base height and the prism height in Session 3. This indicates that the conceptual difficulties experienced by students do not always diminish their motivation to learn.

A high level of engagement was also evident in subject DZ, who actively asked questions and remained focused throughout the learning process. However, this engagement was not fully reflected in the test results. In some answers, DZ still provided incomplete information. When asked about this, DZ stated that, in his view, writing down all known information in full was not particularly important in the problem-solving process. This indicates that although DZ was actively engaged during the learning process, his understanding of the importance of systematically writing down known information when solving problems has not yet been fully developed. These findings highlight the need for teachers to cultivate the habit among students so that they are not only active during the learning process but also able to systematically articulate information and solution steps when working on problems.

In contrast to these two students, the observation results showed that subject RB tended to be less active during the learning process. RB also appeared less engaged in group work activities and had not yet demonstrated optimal contributions during discussions. When interviewed about his involvement during learning, RB stated that he did not particularly like mathematics. This aligns with the subject teacher's statement that RB's academic achievement in mathematics remains relatively low compared to his classmates. Findings from these observations and interviews indicate that RB's low engagement and interest in learning are factors that must be considered when interpreting his learning outcomes. Thus, RB's N-Gain score of 0 does not necessarily indicate persistent misconceptions but may be related to the student's low engagement during the learning process.

Overall, the implementation results show that TopWorksheets-based instructional materials integrated with AR support the improvement of mathematical thinking skills, as evidenced by an increase in scores and a moderate average N-Gain, as well as changes in students' response patterns indicating a shift from misconceptions toward more accurate conceptual understanding, particularly in the "conjecturing" and "convincing" indicators. Nevertheless, there are variations in achievement among individuals that need to be considered as material for reflection and improvement during the Evaluation stage.

Evaluation Phase

The evaluation phase was conducted using both formative and summative methods to assess the quality of the instructional materials based on the results of their development and implementation. Overall, the instructional materials achieved a validity score of 84.31% (highly valid), a practicality score of 86.72% (highly practical), and an average N-Gain of 0.47 (moderate category), indicating an improvement in students' mathematical thinking skills.

The evaluation results also revealed several aspects that still need improvement. Misconceptions regarding the base height and prism height recurred in several groups, necessitating reinforcement through more explicit visualizations and comparison activities. In addition, procedural errors were still found in the application of formulas and calculations, necessitating more gradual and systematic practice. Conjecturing and convincing skills have not yet developed evenly, as some groups have not yet completed these stages of reasoning. Furthermore, variations in student engagement indicate that non-cognitive factors also influence individual achievement.

Based on these results, the instructional materials were found to be valid, practical, and moderately effective in supporting the improvement of mathematical thinking skills. Future

improvements should focus on strengthening students' understanding of the heights of bases and prisms, procedural practice, simplifying the conjecturing and convincing stages, and adjusting the allocation of time and teacher guidance for activities that require mathematical reasoning.

Discussion

Product Quality

The TopWorksheets-based digital instructional materials integrated with AR met the validity criteria at 84.31% and the practicality criteria at 86.72%. These findings are consistent with previous research showing positive acceptance of AR-based instructional materials in geometry learning (Hwang et al., 2023; Koparan et al., 2023; Lestari et al., 2022). However, the overall percentages do not fully reflect variations across aspects, particularly the language aspect, which still requires minor revisions. High practicality also does not directly indicate learning effectiveness. Thus, the product has met the criteria for use in a pilot study but still requires refinement, particularly regarding the language aspect and supporting instruments.

Achievements in Mathematical Thinking Skills

The results of the implementation showed an improvement in mathematical thinking skills, with an average N-Gain of 0.47 in the moderate category, while the average posttest score reached 56.29. For each indicator, the N-Gain fell within the moderate category, with the highest value in generalizing (0.55) and the lowest in specializing (0.43). This pattern suggests that the use of instructional materials combining AR visualization and interactive activities can support students in observing, comparing, and discovering relationships among various prism shapes. However, the activities of conjecturing and convincing require a more explicit reasoning process and justification of reasons, so they are not sufficiently supported by visualization alone. These findings align with those of Kaplar et al. (2021) regarding the support provided by interactive instructional materials for mathematical reasoning, as well as those of Khairunnisa et al. (2022) and Firmasari et al. (2022) regarding the importance of reasoning and conceptual understanding in mathematical proofs.

Persistent Misconceptions About Prisms

A key finding during the Implementation phase was the confusion between the base height and the prism height observed in more than one group and session. This finding aligns with Chiphambo and Mtsi (2021), who reported similar conceptual errors in geometry learning. In this study, the use of AR has not yet fully addressed these difficulties because the visualizations used display the three-dimensional shape of the prism in general but do not provide sufficiently explicit visual distinctions between the base height and the prism height. This situation indicates that the presence of three-dimensional representations does not always guarantee an understanding of every geometric component. Therefore, future developments need to provide clearer cues, such as labels, colors, or comparative displays, as well as guiding questions that direct students to identify and explain relevant measurements.

The Role of AR and TopWorksheets

AR and TopWorksheets were used together as a single integrated product; therefore, this study was unable to determine the contribution of each component to the improvement in mathematical thinking skills. To distinguish these contributions, a comparative design would be required for example, a group using instructional materials without AR. Additionally, variations in achievement among students indicate that responses to the product were not entirely uniform. Although student engagement appeared to be related to achievement in some cases, this study did not use a validated engagement instrument; therefore, this relationship cannot yet be empirically confirmed. These findings are consistent with those of Elsayed and Al-Najrani (2021), who showed that AR can support learning motivation, but that students' active engagement can vary.

Limitations of the Study

This study has several limitations. First, the study involved 19 students from a single school, so the findings have limited generalizability. Second, the absence of a comparison group means that the increase in pretest-posttest scores cannot be specifically attributed to the use of the product. Third, the implementation, which took place over three sessions, limits conclusions regarding the long-term results of using the product. Fourth, the use of the same pretest and posttest instruments allows for the possibility of a familiarity effect regarding the questions. Fifth, working in groups allows for peer assistance among students, which may influence individual results. Sixth, the researcher's involvement in the learning process may influence the students' learning experiences. Seventh, the contributions of AR and TopWorksheets cannot be separated because both were implemented simultaneously. Therefore, future research could use a larger sample and involve multiple schools, employ a design with a comparison group, have a longer implementation duration, and utilize validated engagement instruments. A design comparing the use of instructional materials with and without AR could also be used to examine the contributions of each component.

Overall, the research findings indicate that the integration of AR into TopWorksheets-based digital instructional materials has the potential to support geometry learning, particularly through visualization and interactive activities, as reported in previous studies on AR and mathematics learning (Del Cerro Velázquez & Morales Méndez, 2021; Aditya et al., 2022). This is evident from the level of validity and practicality of the product, as well as the improvement in mathematical thinking skills following the use of the instructional materials. The findings regarding misconceptions about the base height and the prism height also provide important information for the development of future instructional materials so that the presentation of prism concepts becomes clearer and easier for students to understand.

CONCLUSION

This research and development project produced TopWorksheets-based digital instructional materials integrated with Augmented Reality (AR) on the topic of prisms using the ADDIE model. The product achieved a validity percentage of 84.31% and a practicality percentage of 86.72%. Implementation demonstrated an improvement in students' mathematical thinking skills, with an overall N-Gain of 0.47 in the moderate category. Analysis by indicator showed that all four dimensions of mathematical thinking improved within the moderate category, with the highest N-Gain observed in generalizing (0.55), followed by conjecturing (0.50), convincing (0.44), and specializing (0.43). These results indicate that improvement occurred across all indicators, although the magnitude of change differed among them.

Qualitative findings showed that some students developed more accurate conceptual understanding, particularly in making conjectures and providing mathematical justification. However, difficulties remained in distinguishing between the height of the base and the height of the prism, and some students still required stronger support in articulating systematic reasoning during conjecturing and convincing activities. Therefore, the findings provide preliminary evidence that AR-integrated TopWorksheets can support the development of students' mathematical thinking skills in prism learning.

These findings should be interpreted in light of the study's limitations, including the small sample of 19 students from a single school, the use of a one-group pretest-posttest design without a comparison group, and the short implementation period of three sessions. Consequently, the observed improvement cannot be attributed exclusively to the instructional materials. Future research should involve larger and more diverse samples, multiple schools, longer implementation periods, and a comparison or control group to obtain stronger evidence regarding the contribution of AR-integrated TopWorksheets to students' mathematical thinking skills. Future product development should also provide clearer visualization of the distinction between base height and prism height, strengthen scaffolding for conjecturing and

convincing, and include additional exercises to improve procedural accuracy and mathematical justification.

RECOMMENDATION

Future development and implementation of the AR-integrated TopWorksheets should focus on strengthening conceptual scaffolding for prism geometry, particularly in distinguishing between the height of the base and the height of the prism, which remained a recurring source of misconception during the implementation. The instructional materials should also provide more explicit visual cues, guided comparison activities, and additional practice for conjecturing and convincing so that students are not only able to identify and generalize geometric patterns but also justify their mathematical reasoning more systematically. For further research, the product should be tested with larger and more diverse samples across multiple schools, using a comparison or control group and a longer implementation period to obtain stronger evidence regarding its contribution to students' mathematical thinking skills. Future studies are also recommended to examine the separate contributions of Augmented Reality and TopWorksheets and to use validated instruments for measuring student engagement and other non-cognitive factors that may influence learning outcomes.

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AUTHOR CONTRIBUTIONS STATEMENT

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Choirunisa	✓	✓	✓	✓	✓	✓		✓	✓	✓			✓	
Ely Susanti	✓	✓		✓		✓		✓	✓	✓	✓	✓		✓
Cecil Hiltrimartin	✓		✓	✓			✓	✓		✓	✓		✓	

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

ETHICAL APPROVAL

The research related to human use has complied with all the relevant national regulations and institutional policies in accordance with the tenets of the Helsinki Declaration and has been approved by the authors' institutional review board or equivalent committee.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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