



Mathematical Reasoning Processes of Senior High School Students in Solving a Contextual Geometry Problem

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Abstract

Mathematical reasoning is a crucial competency in geometry, yet many students struggle to bridge the gap between abstract concepts and real-world contexts. This study aims to analyze the mathematical reasoning processes of senior high school students when solving a contextual geometry problem. This study involved 36 eleventh-grade students at a senior high school in Central Java, selecting 3 subjects through purposive sampling for analysis. The supporting instruments used included a mathematical geometry reasoning test and an in-depth interview guide. The instruments used have gone through the stages of validity testing from experts. The approach used in this study is descriptive qualitative. The results showed that each subject had a tendency to have a relatively similar mathematical reasoning process but in S1 subjects made a unique representation because made an image first. This finding is of concern to teachers or practitioners that representation is an important part of the mathematical reasoning process, especially in geometry materials.

Keywords: Mathematical reasoning; Contextual geometry problems; Mathematical representation; Problem-solving strategies; Senior high school students

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INTRODUCTION

Mathematics is a subject that is consistently taught at every level of education (Rohati et al., 2023). Students exhibit varying responses when presented with mathematical problems. Some are capable of solving them, while others wish to answer but do not understand the method required to do so (Setiyani et al., 2024). Mathematical reasoning is not only an alternative when formulas are not available but also supports the entire problem-solving process (Ødegaard et al., 2024). Reasoning serves as an essential and inseparable component within mathematics learning (Niswah & Qohar, 2020). Furthermore, mathematics learning standards worldwide have recognized mathematical reasoning as a relevant skill for executing mathematical procedures (Smit et al., 2019). This perspective is supported by (Herbert & Williams, 2023), who state that mathematical reasoning plays a crucial role in mathematics learning, such as in developing students' capacity to comprehend mathematics (Hjelte et al., 2020). This is because the structure of mathematical reasoning is built upon rules and various discourses within mathematics (Aksu & Zengin, 2022), given that the primary difficulty students encounter when reading a task is understanding the problem and its context (Smit et al., 2022).

Mathematical reasoning is defined as a process of reaching conclusions based on gathered facts (Besar, 2018; Kartono & Shora, 2020; Nhiry et al., 2023). Another benefit of mathematical reasoning is its utility in assisting students with mathematical problem-solving

(Jablonski, 2023). When students are unable to develop mathematical reasoning, the meaning of learning degenerates into a mere routine problem-solving procedure (Rohaeti et al., 2020).

In Indonesia, mathematical reasoning has become a component of university entrance examinations, serving as evidence of the immense importance of mathematical reasoning in education. Various studies also demonstrate that contemporary mathematics learning has transformed to place greater emphasis on mathematical reasoning as a vital element (Aisyah et al., 2023). However, expectations do not align with the reality on the ground. Students' mathematical reasoning is classified as low. This is evident from Indonesia's PISA results, where the majority of students perform at a low level (OECD, 2023), despite reasoning being the core of mathematical literacy (Berge & Solberg, 2026). Enhancing students' mathematical reasoning will positively impact their performance in mathematics learning and lead to an increased application of mathematical knowledge to the real world (Mukuka et al., 2021). To enhance mathematical reasoning, researchers, teachers, and practitioners must understand how the construction of mathematical reasoning is formed (Özdemir, 2022). Previous research regarding the conceptual model of mathematical reasoning developed by (Jeannotte & Kieran, 2017) revealed that the process of mathematical reasoning is divided into five parts: generalizing, conjecturing, identifying a pattern, comparing, and classifying. Additionally, (Alderton & Pratt, 2025) conducted a study aiming to reveal the construction of mathematical reasoning as a pedagogical object, concluding that mathematical reasoning is often confined to merely understanding specific procedures. One of the topics taught in mathematics is geometry (Risnawati et al., 2019). Geometry demands high-level mathematical reasoning (Dhlamini et al., 2019). If students' mathematical reasoning is inadequate, they will experience difficulties in interpreting figures in geometry (Urhan & Bülbül, 2023).

Although geometry is one of the important materials taught at the school level, the facts explain that students' mastery of geometry material is still low (Georgiou et al., 2021). One of the factors that causes the low mastery of materials in geometry is due to poor mathematical reasoning (Kyeremeh et al., 2026). In addition, the problems raised in geometry are often related to real life. Contextual problems are problems that are directly related to everyday events (Kolar & Hodnik, 2021). Contextual problems can be used to trigger students' mathematical reasoning processes (Sari & Darhim, 2020). However, further research is needed on how students' mathematical reasoning process in solving contextual problems related to geometry.

This study contributes to the literature on the process of mathematical reasoning in geometry problems. The findings regarding the process of mathematical reasoning can enrich theoretical studies on how students' thinking processes are formed when solving contextual geometry problems. Furthermore, the results of this study are expected to serve as a reference for future researchers and educational practitioners in designing more effective learning strategies to develop students' mathematical reasoning. Specifically, this study addresses the following research questions: What is the mathematical reasoning process of students in solving geometry problems?

METHOD

Design Research

This study uses a qualitative descriptive approach. This approach was chosen because it aims to describe in depth and systematically the phenomenon of students' mathematical reasoning in solving geometry problems, according to the natural conditions that occur in the field (Creswell, 2020). In contrast to research that aims to test hypotheses or build new theories, the descriptive approach emphasizes more on the presentation of facts, characteristics, and relationships between phenomena as they are without intervening or manipulating variables.

The descriptive approach is particularly appropriate for this study because the process of mathematical reasoning is a complex and dynamic cognitive phenomenon, which is strongly influenced by the way students construct their knowledge while solving problems. Therefore,

this study aims to photograph in detail how the mathematical reasoning process takes place, how students demonstrate their reasoning skills, and how the relationship between aspects of reasoning arises during geometry problem solving. Data collection is carried out carefully and a thorough analysis of all the information obtained, resulting in an accurate and in-depth description of the characteristics of students' mathematical reasoning. The results of this study are expected not only to provide an empirical picture of the mathematical reasoning process of students, but also to be useful information for the development of more effective learning strategies in facilitating the improvement of students' mathematical reasoning skills.

Participants

This study involved 36 students who were given a test, from whom 3 subjects were selected in accordance with the data requirements and research objectives. The selection of subjects was carried out progressively using a purposive sampling technique, which involves selecting participants based on the needs of developing categories and the theory being constructed. This is in line with the targeted sampling strategy commonly used in case studies, where participants are selected for their potential in providing in-depth and information-rich data (Yin, 2018). After the data from the initial subjects were analyzed, the researcher determined the subsequent subjects who were expected to enrich, deepen, or confirm the emerging categories. This condition indicates that the research had reached data saturation. Consequently, the number of subjects in this study was not fixed in advance and was determined by the need to obtain comprehensive and detailed data to achieve the research objectives. In line with the descriptive qualitative approach employed, this study did not use a saturation table, as the focus was on obtaining a thorough and in-depth description of students' mathematical reasoning processes.

Research Instruments

An instrument is a measurement tool used in research. Qualitative research positions the researcher as the primary instrument. Meanwhile, other supporting instruments remain necessary. In this study, the supporting instruments used were a mathematical reasoning test and in-depth interviews. The validation of the mathematical reasoning test questions used in this study is content validation. This process is assessed logically by experts in their fields (expert judgment). The experts referred to in this study involve 1 professor in the field of mathematics education and 1 assistant professor in the field of mathematics education. After the test instrument is declared valid, it can be used to obtain data about the student's mathematical reasoning process. The language used in the test instrument was Bahasa Indonesia. The test and interview instruments were employed to explore information regarding the processes of students' mathematical reasoning in solving geometry problems. The test instrument is presented in Table 1.

Table 1. Mathematical reasoning test instruments

Geometry Test
Sebuah gunung nampak dari atas dengan posisi berada pada koordinat pusat (0,0). Gunung itu merupakan salah satu gunung berapi yang masih aktif. Data menunjukkan bahwa gunung tersebut telah meletus sebanyak 4 kali sejak tahun 2000. Banyaknya korban dipengaruhi oleh jumlah pemukiman warga yang memang dekat dengan gunung berapi tersebut. Setidaknya ada 5 desa yang berada di sekitar gunung berapi dengan koordinat sebagai berikut : Desa krinjing (3, 2); Desa Klakah (-4,-3); Desa Tlogolele (-5,5); Desa Balerante (2,-4); Desa Jarakah (6,1). Dalam catatan data, menunjukkan bahwa gunung berapi tersebut pernah mencatatkan jarak terjauh lahar panas dengan radius sejauh 5 km. Jika mengacu pada data tersebut maka kelompokkanlah desa yang paling aman dan berikan saran tentang jarak koordinat teraman pembangunan desa jika seandainya diperlukan.

Data Analysis

Data analysis in this study was conducted interactively and continuously from the data collection process until the completion of the research. The data analysis technique refers to

the model by (Miles & Huberman, 1994), which encompasses three main stages: data reduction, data display, and conclusion drawing/verification. These three stages were executed in a cyclical manner so that data analysis continuously evolved alongside the accumulation of data obtained in the field.

The first stage of qualitative data analysis is data reduction. Rather than treating all collected material with equal weight, the researcher systematically selects, focuses, simplifies, and abstracts the raw information gathered from the geometry problem-solving tests, in-depth transcripts, and observational field notes. This meticulous filtering process involves discarding irrelevant details and Identify patterns from students' mathematical reasoning processes in solving geometry problems.

The second stage is data display. The reduced data were then presented systematically in the form of descriptive narratives, tables, diagrams, and conceptual models depicting the relationships between categories. The purpose of data display is to facilitate the researcher in understanding emerging patterns, identifying connections between categories, and visualizing the construction process of students' mathematical reasoning while solving geometry problems. The data display was also supplemented with excerpts from interviews and student work as empirical evidence supporting each discovered category.

The third stage is conclusion drawing and verification. Conclusions were formulated based on patterns, relationships, and categories obtained during the data analysis process. Conclusion drawing was performed gradually and continuously verified through cross-data comparisons, discussions between categories, and re-examination of raw data to ensure that the constructed theory was truly rooted in empirical data. The verification process was carried out continuously throughout the study until a substantive theory capable of comprehensively explaining the processes of students' mathematical reasoning in geometry problem-solving was obtained.

RESULTS AND DISCUSSION

This section presents the results of the analysis of data obtained from student work and in-depth interviews. The analysis was conducted progressively using the Grounded Theory approach so that the presented categories represent constructions emerging from empirical data. Each category is explained based on evidence found in the field and subsequently interpreted to explain the construction process of students' mathematical reasoning in geometry problem-solving. Out of the 36 students who completed the mathematical reasoning test, we selected 3 students as the initial data source to analyze the mathematical reasoning process. These three students were subsequently coded as S1, S2, and S3.

Subject S1

At this stage, S1 subjects have been given a mathematical reasoning test. The following is a documentation of the results of the S1 subject's answers.

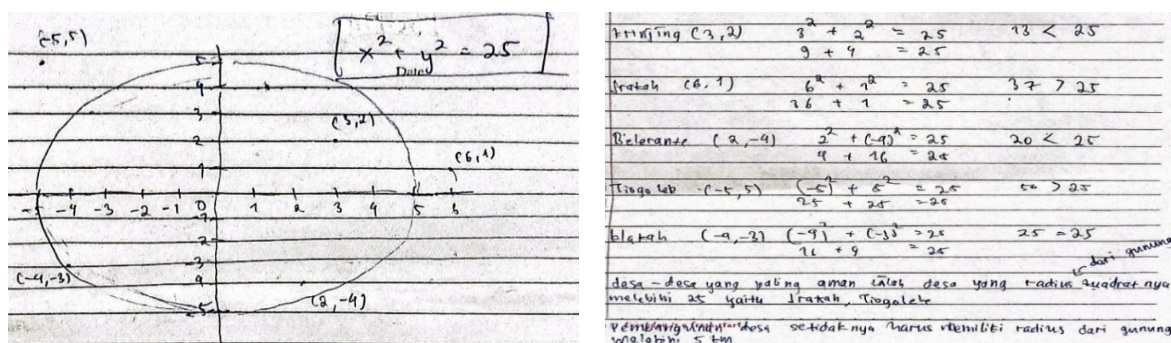


Figure 1. Documentation of the S1 subject answer sheet

Based on Figure 1, subject S1 first wrote down the circle equation $x^2 + y^2 = 25$ and proceeded to draw a circle centered at (0, 0). Afterward, subject S1 wrote down all village

coordinates according to the information provided in the problem. These activities were then analyzed through the open coding process to identify the actions performed by subject S1 while solving the geometry problem. The findings from the test results were also supported by the interview transcript of subject S1. The following is the transcript of the interview with subject S1.

- R : On this answer sheet, why did you make a circle equation $x^2 + y^2 = 25$?
- S1 : Because the center at (0, 0) I remember that the circle equation in the center (0, 0) means to use it $x^2 + y^2 = r^2$
- R : Why are you drawing a circle here?
- S1 : So that I know which village position is within or outside the radius.
- R : Why do all your village points be placed in the coordinate field?
- S1 : Because the coordinates have been given in the question, so I just have to place it so that it can be seen in its position.
- R : What are the benefits after all the points are drawn?
- S1 : I can see which ones are inside the circle and which ones are outside.
- R : What this is the calculation for $3^2 + 2^2$?
- S1 : To find out if the point meets the circle equation.
- R : Why do you compare the results with 25?
- S1 : Because 25 is the square of the radius. If the result is less than 25, it means that the point is still in the circle. If it is the same, it means that it is right in the circle. If it is bigger, it means that it is already outside.
- R : Where do you know the concept?
- S1 : From the material that has been studied.
- R : Why is there a comparison of $13 < 25$?
- S1 : To find out the position of the village whether it is in a safe area or not.
- R : Why are only Jarakah and Tlogolele Villages safe?
- S1 : Because the result of the calculation is greater than 25, it is located outside the circle. That means that Jarakah and Tlogolele Villages are not affected by the larval burst.
- R : At the end you write that the village development should be more than 5 km from the mountain. Why?
- S1 : Because all points that are more than a radius of 5 km are outside the hazardous area.

Based on the results of the tests and interviews that have been presented in the previous section, the next analysis step taken is to enter the open coding stage. This process aims to identify, name, and categorize the initial concepts that emerge directly from the raw data that has been collected. To provide a clearer understanding of the analysis of subject S1's written responses and interview results, the findings are presented in Table 2.

Table 2. Process of Mathematical Reasoning Subject S1

Test results	Interview	Process of Math. Reasoning
Write a circle equation $x^2 + y^2 = 25$	I remember that the circle equation in the center (0, 0) means that it is $x^2 + y^2 = r^2$	Conjecturing
Drawing a circle	So that I know which village positions are within or outside the radius.	Representation
Putting the coordinates of the village	Because the coordinates have been given in the question, so I just have to place it so that it can be seen in its position.	Identifying a pattern
Counting $3^2 + 2^2$	To find out if the point meets the circle equation.	Generalizing

Test results	Interview	Process of Math. Reasoning
Compare with 25	Because 25 is the square of the radius. If the result is less than 25, it means that the point is still in the circle. If it is the same, it means that it is right in the circle. If it is bigger, it means it is outside.	Comparing
Write $13 < 25$	To find out the position of the village whether it is in a safe area or not.	Classifying
Write $37 > 25$	To find out the position of the village whether it is in a safe area or not.	Classifying
Defining a safe village	Because the result of the calculation is greater than 25, it is located outside the circle. That means that Jarakah and Tlogolele Villages are not affected by the larval burst.	Classifying
Providing village development advice	Because all points that are more than a 5 km radius are outside the hazardous area.	Generalizing

Subject S2

At this stage, S2 subjects have been given a mathematical reasoning test. The following is the documentation of the results of the S2 subject answers.

Pengelompokan desa berdasarkan tingkat keamanan

Berdasarkan data radius aliran bahar panas sejauh 5 km, desa-desa dikelompokkan menggunakan perhitungan jarak koordinat ($d = \sqrt{x^2 + y^2}$) sebagai berikut:

kelompok Desa aman (Jarak > 5 km)

Tlogolele : memiliki koordinat $(-5, 5)$ dengan jarak kurang lebih 7,07 km dari pusat gunung.

Jarakah : memiliki koordinat $(6, 1)$ dengan jarak kurang lebih 6,08 km dari pusat gunung.

kelompok desa beresiko (Jarak ≤ 5 km)

Klalah : memiliki koordinat $(-4, -3)$ dengan jarak 5 km

Balerante : memiliki koordinat $(2, -4)$ dengan jarak 4,47 km

Krinjing : memiliki koordinat $(3, 2)$ dengan jarak 3,61 km

Saran Jarak koordinat teraman pembangunan desa

Jarak minimal : lebih dari 5 km

koordinat : (x, y) harus $x^2 + y^2 > 25$

Figure 2. Documentation of the S2 subject answer sheet

Figure 2 demonstrates that subject S2 initiated problem-solving by grouping the villages based on their safety levels. Distinct from subject S1, who first constructed a visual representation in the form of a circle on a coordinate plane, subject S2 immediately wrote down the safety level of each village. Subsequently, the subject calculated the distance of each village

from the center of the mountain and then grouped them into two categories: safe villages (distance > 5 km) and at-risk villages (distance ≤ 5 km). In the final section, the subject provided a recommendation that the location for village development should have a distance of more than 5 km from the center of the mountain or satisfy the condition $x^2 + y^2 > 25$. The findings from the test results were also supported by the interview transcript of subject S2. The following is the transcript of the interview with subject S2.

- R : Before you start working, what is the first thing you understand from the problem?
 S2 : I understand that I have to determine which villages are safe and which villages are still at risk based on their distance from the mountain.
 R : Where do you know that distance is important?
 S2 : Because in the question it was explained that the radius of the hot lava flow is 5 km, so I have to compare the distance of each village with that radius.
 R : Why do you write down the formula right away $d = \sqrt{x^2 + y^2}$?
 S2 : Because all villages already have their coordinates and the center of the mountain is at point (0.0). So I think the easiest way is to calculate the distance of each village to the center of the mountain.
 R : Why don't you draw the circle?
 S2 : I'm more used to using formulas.
 R : Why do you count distances one by one?
 S2 : So that I know the actual distance from each village to the center of the mountain. After that, it can only determine whether the village is safe or not.
 R : How do you define a safe village?
 S2 : If the result is more than 5 km, I will put it in the safe village group.
 R : Why use the 5 km limit?
 S2 : Therefore, the limit of the radius of hot lava flow given to the problem.
 R : Then how to determine the village that is at risk?
 S2 : If the distance is less than or equal to 5 km, it means that it is still in the affected area.
 R : At the end of the section you write that the coordinates of village development must meet $x^2 + y^2 > 25$. Why?
 S2 : Because if the distance is more than 5 km, it means that the square distance must also be more than 25. So the coordinate requirements are like that.
 R : How did you get these conditions?
 S2 : From the distance formula. If $d > 5$, means that after being squared into $x^2 + y^2 > 25$.

Based on the results of the tests and interviews that have been presented in the previous section, the next analysis step taken is to enter the open coding stage. This process aims to identify, name, and categorize the initial concepts that emerge directly from the raw data that has been collected. The results of open coding data for S2 subjects are presented in Table 3.

Table 3. Process of Mathematical Reasoning Subject S2

Test results	Interview	Process of Math. Reasoning
S2 subjects start solving problems by grouping villages based on security level	I understand that I have to determine which villages are safe and which villages are still at risk based on their distance from the mountain.	Classifying
Write down the radius of hot lava as far as 5 km	So that I know which village positions are within or outside the radius.	Identifying a pattern

Test results	Interview	Process of Math. Reasoning
Write down the formula $d = \sqrt{x^2 + y^2}$	Because the coordinates have been given in the question, so I just have to place it so that it can be seen in its position.	Conjecturing
Write down the group of safe villages with a radius of > 5 km	To find out if the point meets the circle equation.	Classifying
Conducting coordinate point testing in each village	Because 25 is the square of the radius. If the result is less than 25, it means that the point is still in the circle. If it is the same, it means that it is right in the circle. If it is bigger, it means it is outside.	Comparing
Writing down suggestions for safe village development	Because if the distance is more than 5 km, it means that the square distance must also be more than 25. So the coordinate requirements are like that.	Generalizing

The results show that the initial activities of S2 subjects began with identifying the goal of solving problems, namely grouping villages based on the level of security. Next, the subject chooses a mathematical model in the form of a distance formula $d = \sqrt{x^2 + y^2}$ to calculate the distance of each village to the center of the mountain. The results of the calculation were then compared with a danger radius of 5 km as the basis for classifying villages into safe and risky categories. In the final stage, the subject drafts a general rule that the village construction site must be at a distance of more than 5 km from the center of the mountain or meet the requirements $x^2 + y^2 > 25$.

Subject S3

At this stage, S3 subjects have been given a mathematical reasoning test. The following is the documentation of the results of the S3 subject answers.

pusat gunung di koordinat (0,0)	
Dato koordinat desa	rumus $\Rightarrow$ jarak = $\sqrt{x^2 + y^2}$
• Kiriang (3,2) • Klakah (-4,3) • Jarak (6,1) • Tlogolele (-5,5) • Balekambang (2,-4)	
1. Kiriang (3,2)	
$\sqrt{3^2 + 2^2} = \sqrt{13} = 3,6 \text{ km} \rightarrow$ tidak aman	
2. Klakah (-4,3)	
$\sqrt{16 + 9} = \sqrt{25} = 5 \text{ km} \rightarrow$ batas aman (rawan)	
3. Jarak (6,1)	
$\sqrt{36 + 1} = \sqrt{37} = 6,1 \rightarrow$ aman	
4. Tlogolele (-5,5)	
$\sqrt{25 + 25} = \sqrt{50} = 7,1 \text{ km} \rightarrow$ aman	
5. Balekambang (2,-4)	
$\sqrt{4 + 16} = \sqrt{20} = 4,5 \text{ km} \rightarrow$ tidak aman	
kesimpulan = semakin jauh dari pusat gunung (semakin) (lebih dari 5 km) semakin aman	

Figure 3. Documentation of the S3 subject answer sheet

Figure 3 shows that the S3 subject begins the completion by writing down the known information, namely the center of the mountain is at the coordinates (0.0) and the list of coordinates of the entire village. Next, the subject writes down the distance formula $\sqrt{x^2 + y^2}$

as a mathematical model to determine the distance of each village to the center of the mountain. After that, the subjects calculated the distance of each village individually, then compared the results with a 5 km danger radius to determine the safety status of each village. At the end, the subjects concluded that villages that were more than 5 km from the center of the mountain were safe, while villages that were less than or equal to 5 km were considered unsafe. The findings of the test results are also supported by interview transcripts from S3 subjects. The following are the transcripts of interviews with S3 subjects.

- R : What was the first thing you did?
 S3 : I first wrote the center of the mountain at the coordinates (0,0) and all the coordinates of the village in the question so that the information is clearer.
 R : Why do you feel the need to rewrite this information?
 S3 : So that I don't go back and forth to see the problem. If all the information is written down, it is easier for me to calculate one by one.
 R : Why did you write the center of the mountain first?
 S3 : Since all distances are calculated from the center of the mountain, so that's the reference point.
 R : After writing down the information, why do you use the distance formula $\sqrt{x^2 + y^2}$?
 S3 : Because what is asked is the distance from the village to the center of the mountain. I remember that the distance from point (0,0) to a point can be calculated using that formula.
 R : Why don't you make the drawing?
 S3 : In my opinion, it is faster if it is calculated directly using formulas.
 R : Why did you count all the villages one by one?
 S3 : Because each village has different coordinates, so the distance is also different. I have to count everything to know which ones are safe.
 R : How do you determine whether a village is safe or not?
 S3 : I compared the results of the distance with 5 kilometers.
 R : Why use the number 5?
 S3 : Because in question it was explained that the radius of the hot lava flow was 5 km.
 R : Why did you write "safe limits"?
 S3 : Because the result is exactly 5 km. So I think the village is right on the boundary of the area that can still be affected.
 R : At the end you write that the farther away from the center of the mountain, the safer it will be. How did you come to that conclusion?
 S3 : After I saw all the calculations, the villages that were more than 5 km away were all safe. So I concluded the further away from the center of the mountain, the safer it became.

Based on the results of the tests and interviews that have been presented in the previous section. This process aims to identify, name, and categorize the initial concepts that emerge directly from the raw data that has been collected. The results of open coding data for S3 subjects are presented in Table 4.

Table 4. Process of Mathematical Reasoning Subject S3

Test results	Interview	Process of Math. Reasoning
Write the center of the mountain at the coordinates (0,0).	I first wrote the center of the mountain at the coordinates (0,0) and all the coordinates of the village in the question so that the information is clearer.	Identifying a pattern
Write down a list of coordinates of the entire village.	So that I don't go back and forth to see the problem. If all the	Classifying

Test results	Interview	Process of Math. Reasoning
Write down the distance formula $\sqrt{x^2 + y^2}$	information is written down, it is easier for me to calculate one by one. Because what is asked is the distance from the village to the center of the mountain. I remember that the distance from point (0,0) to a point can be calculated using that formula.	Conjecturing
Using a formula to calculate the distance of Krincing Village.	Because each village has different coordinates, so the distance is also different. I have to count everything to know which one is safe.	Generalizing
Using a formula to calculate the distance of Klakah, Jrakah, Tlogolele, Balerante Villages	Because each village has different coordinates, so the distance is also different. I have to count everything to know which ones are safe.	Generalizing
Specify "safe", "unsafe", and "safe limit" statuses.	I compared the results of the distance with 5 kilometers.	Comparing
Writes down the conclusion that the farther away from the center of the mountain (>5 km) the safer it is.	After I saw all the calculations, the villages that were more than 5 km away were all safe. So I concluded the further away from the center of the mountain, the safer it became.	Generalizing

The results showed that the initial activity of the S3 subject began by identifying the shape of the circle equation in the center (0,0) based on the information of the question and then organizing the information by writing a list of coordinates of the entire village. Next, the S3 subject chooses a model or formula to solve the problem. After that, the S3 subjects do calculations until they get the right results and conclusions. Identification of problems is needed so that students can know what plan to use (Jonsson et al., 2022).

The results of the study indicate that all three subjects constructed mathematical reasoning through a relatively similar sequence of processes, namely identifying problem information, selecting mathematical representations, performing mathematical verification, classifying objects, and formulating generalizations. Nonetheless, variations were observed in the representation strategies employed by each subject. Representation assists students in facilitating problem-solving (Mainali, 2021). Subject S1 initiated the solution by constructing a visual representation in the form of a circle drawing and mapping the village coordinates, whereas S2 and S3 preferred using symbolic representation through the distance formula. This finding is supported by (Öz & Işık, 2024), who revealed that representation helps students connect contextual information with mathematical concepts, thereby enabling them to choose appropriate solution procedures. This indicates that mathematical representation constitutes a critical stage bridging problem comprehension with the decision-making process (Minarni et al., 2016). Based on this viewpoint, mathematical reasoning is present in every aspect of mathematics and represents a vital component in the mathematics learning process (Çoban & Tezci, 2022).

These differences in representation strategies indicate that the construction of mathematical reasoning does not develop through a single, identical path. Subject S1 relied more heavily on visual representation to comprehend spatial relationships among objects prior to conducting numerical verification, whereas S2 and S3 utilized direct symbolic modeling via

the distance formula. Engagement in mathematical modeling is made possible through student reasoning (Kularajan & Czoher, 2025). Although the initial strategies utilized differed, all three subjects exhibited a convergent reasoning pattern during the verification, classification, and generalization stages. This finding indicates that variations in representation do not alter the fundamental structure of the reasoning process, but merely influence the manner in which students construct solutions.

The subsequent stage that appeared consistently across all subjects was mathematical verification. After constructing a representation, all subjects verified the position of each village using appropriate mathematical procedures. S1 performed verification by comparing the value $x^2 + y^2$ against 25, while S2 and S3 calculated the Euclidean distance and compared it with the danger radius of 5 km. Although the procedures utilized differed, the objective achieved was identical: determining whether a village was situated inside or outside the danger zone. This finding demonstrates that verification serves as the primary mechanism connecting representation with mathematical decision-making. Verification aims to determine whether a claim is supported or rejected based on a series of evidence (Gong et al., 2026).

Following verification, all subjects classified the villages into the categories of safe, safety limit, or at-risk. This classification process indicates that mathematical reasoning does not terminate at computational activities but progresses toward the interpretation of results. At this stage, students do not merely generate numerical values but also ascribe meaning to those results based on the provided problem context. Consequently, mathematical reasoning encompasses the ability to connect calculation results with real-world situations to produce logical decisions..

The final stage emerging across all subjects was generalization. The three subjects concluded that locations with a distance greater than 5 km from the center of the mountain represent safe areas for village development. This generalization indicates that students have moved from solving specific cases toward formulating rules that apply more generally. Generalization helps to establish connections between the objects being worked upon (Santos et al., 2022). The ability to formulate general rules is an important indicator in mathematical reasoning because it demonstrates that students do not merely solve problems but are also capable of abstracting the underlying mathematical relationships (Son et al., 2020). This finding aligns with research by (Jeannotte & Kieran, 2017), which states that mathematical reasoning develops through the processes of constructing relationships, providing justifications, and drawing conclusions applicable to other situations.

CONCLUSION

Based on the results and research discussions, it can be concluded that in general, the three subjects show consistency in problem-solving patterns as well as characteristics of mathematical reasoning processes that tend to be uniform, especially in terms of conceptual grouping, the use of distance formulas, and threshold-based conclusions. Nevertheless, S1 subjects exhibit unique and distinctive reasoning characteristics through the explicit use of visual representations. S1 subjects consistently begin their cognitive process by creating geometric sketches to concretize spatial positions, which serve as an essential tool in translating abstract data into more intuitive mathematical models. These findings confirm that visual representations play an important role as self-help that facilitates the subject's transition from problem understanding to numerical verification stage.

RECOMMENDATION

Based on these findings, we recommend that future researchers explore the process of students' reasoning in other mathematical topics, and investigate further the most optimal forms of specific learning interventions to accelerate students' transition from the verification stage to the generalization stage. Furthermore, it is expected that practitioners can design and integrate the provision of adaptive and effective scaffolding within the mathematics learning

process to stimulate students' reasoning capabilities. Teachers also need to facilitate the diversity of students' thinking strategies, whether visual or symbolic, as both paths are valid in constructing a comprehensive reasoning structure.

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AUTHOR CONTRIBUTIONS STATEMENT

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Yusuf Arkham Prihandika	✓	✓	✓		✓	✓		✓	✓	✓	✓		✓	
Wardono	✓	✓		✓								✓		
Emi Pujiastuti	✓	✓		✓								✓		
Walid	✓	✓		✓								✓		

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

ETHICAL APPROVAL

The researchers meticulously followed ethical protocols throughout the research process, adhering to the principles outlined in the Declaration of Helsinki.

DATA AVAILABILITY

The authors confirm that the data supporting the findings of this study are available within the article.

REFERENCES

- Aisyah, N., Susanti, E., Meryansumayeka, Siswono, T. Y. E., & Maat, S. M. (2023). Proving geometry theorems: Student prospective teachers' perseverance and mathematical reasoning. *Infinity Journal*, 12(2), 377–392. <https://doi.org/10.22460/infinity.v12i2.p377-392>
- Aksu, N., & Zengin, Y. (2022). Disclosure of students' mathematical reasoning through collaborative technology-enhanced learning environment. *Education and Information Technologies*, 27(2), 1609–1634. <https://doi.org/10.1007/s10639-021-10686-x>
- Alderton, J., & Pratt, N. (2025). The construction of mathematical reasoning as a pedagogical object. *Educational Studies in Mathematics*, 120(2), 299–315. <https://doi.org/10.1007/s10649-025-10420-1>
- Berge, R. L., & Solberg, E. M. (2026). Assessing mathematical reasoning: Analyzing imitative and creative reasoning by students. *Educational Studies in Mathematics*, 122(3), 407–425. <https://doi.org/10.1007/s10649-026-10487-4>
- Besar, P. H. S. N. (2018). Situated learning theory: The key to effective classroom teaching? *HONAI: International Journal for Educational, Social, Political & Cultural Studies*, 1(1), 49–60. <https://www.journals.mindamas.com/index.php/honai/article/view/1022>
- Çoban, H., & Tezci, E. (2022). Mathematical reasoning: Bibliometric analysis of the literature. *OPUS-Journal of Society Research*, 19(45), 88–102. <https://doi.org/10.26466/opusjsr.1062867>
- Creswell, J. W. (2020). *Penelitian kualitatif & desain riset* (Edisi Indonesia). Pustaka Pelajar.
- Dhlamini, Z. B., Chuene, K., Masha, K., & Kibirige, I. (2019). Exploring grade nine geometry spatial mathematical reasoning in the South African annual national assessment. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(11), 1–17. <https://doi.org/10.29333/ejmste/105481>
- Georgiou, Y., Ioannou, A., & Kosmas, P. (2021). Comparing a digital and a non-digital

- embodied learning intervention in geometry: Can technology facilitate? *Technology, Pedagogy and Education*, 30(2), 345–363. <https://doi.org/10.1080/1475939X.2021.1874501>
- Gong, H., Li, J., Wu, J., Liu, Q., & Wu, S. (2026). STRIVE: Structured reasoning for self-improvement in claim verification. *Machine Intelligence Research*, 23(1), 185–199. <https://doi.org/10.1007/s11633-025-1598-5>
- Herbert, S., & Williams, G. (2023). Eliciting mathematical reasoning during early primary problem solving. *Mathematics Education Research Journal*, 35(1), 77–103. <https://doi.org/10.1007/s13394-021-00376-9>
- Hjelte, A., Schindler, M., & Nilsson, P. (2020). Kinds of mathematical reasoning addressed in empirical research in mathematics education: A systematic review. *Education Sciences*, 10(10), Article 289. <https://doi.org/10.3390/educsci10100289>
- Jablonski, S. (2023). Real objects as a reason for mathematical reasoning: A comparison of different task settings. *International Electronic Journal of Mathematics Education*, 18(4), 1–18. <https://doi.org/10.29333/iejme/13859>
- Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, 96(1), 1–16. <https://doi.org/10.1007/s10649-017-9761-8>
- Jonsson, B., Mossegård, J., Lithner, J., & Karlsson Wirebring, L. (2022). Creative mathematical reasoning: Does need for cognition matter? *Frontiers in Psychology*, 12, Article 797807. <https://doi.org/10.3389/fpsyg.2021.797807>
- Kartono, & Shora, R. Y. (2020). Effectiveness of process oriented guided inquiry learning with peer feedback on achieving students' mathematical reasoning capabilities. *International Journal of Instruction*, 13(3), 555–570. <https://doi.org/10.29333/iji.2020.13338a>
- Kolar, V. M., & Hodnik, T. (2021). Mathematical literacy from the perspective of solving contextual problems. *European Journal of Educational Research*, 10(1), 467–483. <https://doi.org/10.12973/eu-jer.10.1.467>
- Kularajan, S., & Czoher, J. A. (2025). The role of reasoning with quantities in undergraduates' modeling activities. *International Journal of Research in Undergraduate Mathematics Education*, 11(3), 622–652. <https://doi.org/10.1007/s40753-025-00273-7>
- Kyeremeh, P., Awuah, F. K., & Orey, D. C. (2026). From cultural awareness to integration: Faculty attempt to incorporate the outside-of-school geometry into geometry teaching. *Science & Education*. <https://doi.org/10.1007/s11191-026-00745-5>
- Mainali, B. (2021). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science and Technology*, 9(1), 1–21. <https://doi.org/10.46328/ijemst.1111>
- Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). SAGE Publications.
- Minarni, A., Napitupulu, E. E., & Husein, R. (2016). Mathematical understanding and representation ability of public junior high school in North Sumatra. *Journal on Mathematics Education*, 7(1), 43–56. <https://doi.org/10.22342/jme.7.1.2816.43-56>
- Mukuka, A., Mutarutinya, V., & Balimuttajjo, S. (2021). Mediating effect of self-efficacy on the relationship between instruction and students' mathematical reasoning. *Journal on Mathematics Education*, 12(1), 73–92. <https://doi.org/10.22342/jme.12.1.12508.73-92>
- Nhiry, M., Abouhanifa, S., & El Khouzai, E. M. (2023). The characterization of mathematical reasoning through an analysis of high school curricula and textbooks in Morocco. *Cogent Education*, 10(1), Article 2188797. <https://doi.org/10.1080/2331186X.2023.2188797>
- Niswah, U., & Qohar, A. (2020). Mathematical reasoning in mathematics learning on pyramid volume concepts. *Malikussaleh Journal of Mathematics Learning (MJML)*, 3(1), 23–26. <https://doi.org/10.29103/mjml.v3i1.2400>
- Ødegaard, R. P., Arnesen, K. K., & Langfeldt, M. B. (2024). Tools to support learning to plan

- discussions that promote students' mathematical reasoning. *Journal of Mathematics Teacher Education*, 29(2), 205–234. <https://doi.org/10.1007/s10857-024-09668-w>
- OECD. (2023). *OECD economic outlook*. OECD Publishing.
- Öz, T., & Işık, A. (2024). Exploring mathematical reasoning skills of middle school students. *Thinking Skills and Creativity*, 53, Article 101612. <https://doi.org/10.1016/j.tsc.2024.101612>
- Özdemir, F. (2022). Examining pre-service teachers' views on reasoning, developing reasoning and importance of reasoning skills in geometry. *Acta Didactica Napocensia*, 15(2), 1–12. <https://doi.org/10.24193/adn.15.2.1>
- Risnawati, Andrian, D., Azmi, M. P., Amir, Z., & Nurdin, E. (2019). Development of a definition maps-based plane geometry module to improve the student teachers' mathematical reasoning ability. *International Journal of Instruction*, 12(3), 541–560. <https://doi.org/10.29333/iji.2019.12333a>
- Rohaeti, E. E., Fitriani, N., & Akbar, F. (2020). Developing an interactive learning model using Visual Basic applications with ethnomathematical contents to improve primary school students' mathematical reasoning. *Infinity Journal*, 9(2), 275–286. <https://doi.org/10.22460/infinity.v9i2.p275-286>
- Rohati, R., Kusumah, Y. S., & Kusanadi, K. (2023). Exploring students' mathematical reasoning behavior in junior high schools: A grounded theory. *Education Sciences*, 13(3), Article 252. <https://doi.org/10.3390/educsci13030252>
- Santos, L., Mata-Pereira, J., da Ponte, J. P., & Oliveira, H. (2022). Teachers' understanding of generalizing and justifying in a professional development course. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(1), 1–14. <https://doi.org/10.29333/ejmste/11488>
- Sari, D. P., & Darhim. (2020). Implementation of REACT strategy to develop mathematical representation, reasoning, and disposition ability. *Journal on Mathematics Education*, 11(1), 145–156. <https://doi.org/10.22342/jme.11.1.7806.145-156>
- Setiyani, Waluya, S. B., Sukestiyarno, Y. L., & Cahyono, A. N. (2024). Construction of reflective thinking: A field independent student in numerical problems. *Journal on Mathematics Education*, 15(1), 151–172. <https://doi.org/10.22342/jme.v15i1.pp151-172>
- Smit, R., Dober, H., Hess, K., Bachmann, P., & Birri, T. (2022). Supporting primary students' mathematical reasoning practice: The effects of formative feedback and the mediating role of self-efficacy. *Research in Mathematics Education*, 24(2), 1–24. <https://doi.org/10.1080/14794802.2022.2062780>
- Smit, R., Hess, K., Bachmann, P., Blum, V., & Birri, T. (2019). What happens after the intervention? Results from teacher professional development in employing mathematical reasoning tasks and a supporting rubric. *Frontiers in Education*, 3, Article 113. <https://doi.org/10.3389/educ.2018.00113>
- Son, A. L., Darhim, & Fatimah, S. (2020). Students' mathematical problem-solving ability based on teaching models intervention and cognitive style. *Journal on Mathematics Education*, 11(2), 209–222. <https://doi.org/10.22342/jme.11.2.10744.209-222>
- Urhan, S., & Bülbül, A. (2023). Analysis of mathematical proving in geometry based on Habermas' construct of rationality. *Mathematics Education Research Journal*, 35(4), 929–959. <https://doi.org/10.1007/s13394-022-00420-2>
- Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE Publications.